

Time variation of G and Λ , acceleration of the universe, coincidence problem and Mach's cosmological coincidence

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Abstract

We study a gravitational model in which *scale transformations* play the key role in obtaining dynamical G and Λ . We take a non-scale invariant gravitational action with a cosmological constant and a gravitational coupling constant. Then, by a scale transformation, through a dilaton field, we obtain a new action containing cosmological and gravitational coupling terms which are dynamically dependent on the dilaton field with Higgs type potential. The vacuum expectation value of this dilaton field, through spontaneous symmetry breaking on the basis of *anthropic principle*, determines the time variations of G and Λ . The relevance of these time variations to the current acceleration of the universe, coincidence problem, Mach's cosmological coincidence and those problems of standard cosmology addressed by inflationary models, are discussed. The current acceleration of the universe is shown to be a result of phase transition from radiation toward matter dominated eras. No real coincidence problem between matter and vacuum energy densities exists in this model and this apparent coincidence together with Mach's cosmological coincidence are shown to be simple consequences of a new kind of scale factor dependence of the energy momentum density as $\rho \sim a^{-4}$. This model also provides the possibility for a super fast expansion of the scale factor at very early universe by introducing exotic type matter like cosmic strings.

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1 Introduction

The question of varying gravitational “constant” has been among the most controversial issues in fundamental physics. It raised by Dirac who introduced the large number hypothesis [1], and has recently become a subject of intensive experimental and theoretical studies [2]. Modern theories, like the string/M-theory or brane models do not necessarily require such a variation but they provide a natural and self consistent framework for such variations by assuming the existence of additional dimensions. Time variation of couplings in these multidimensional theories has also recently been studied and their consistency with the available observational data for distant Type Ia supernovae has been analyzed in [3]. It was shown that in these models a small variation of gravitational coupling arises that makes distant supernovae to appear brighter, in contradiction with recent observations of high z supernovae. However, due to the fact that the magnitude of the effect is not large enough, one could not safely discard these multidimensional models. For example, a positive rate of variation $\frac{\dot{G}(t)}{G(t)}$ has been predicted within a $N = 1$ ten-dimensional supergravity and with non-dynamical dilaton [4].

There are also other models in which the time variation of couplings is generated by the dynamics of a cosmological scalar (dilaton) field. For example, Damour *et al* have constructed a generalized Jordan-Brans-Dicke model in which the dilaton field couples with different strengths to visible and dark matter, and provided nontrivial bounds on the coupling constants of this field to matter [5]. On the other hand, Bekenstein [6] and Bertolami [7], have introduced models in which both gravitational coupling and cosmological term are time dependent. Of particular interest for us is the Bertolami’s results in that the gravitational coupling and cosmological term respectively behave like $G \sim t$, $\Lambda \sim t^{-2}$ for one solution, and $G \sim t^2$,

$\Lambda \sim t^{-2}$ for another one.

Conformal invariance, on the other hand, has played a key role in the study of G and Λ varying theories. Bekenstein was the first who introduced this possibility and tried to resolve G -varying problem [6]. Conformal invariance implies that the gravitational theory is invariant under local changes of units of length and time. These local transformations relate different unit systems or conformal frames via space time dependent conformal factors, and these unit systems are dynamically distinct. This fact leads to variability of the fundamental constants. Recently, it is shown that one can use this dynamical distinction between two unit systems usually used in cosmology and particle physics to construct a cancelation mechanism which reduces a large cosmological constant to a sufficiently small value, and study the effects of this model both on the early and late time asymptotic behavior of the scale factor in the standard cosmological model. The idea that gravitational coupling may be the result of a spontaneous conformal symmetry breaking and its relevance to Mach principle is also studied in [8].

Unlike above models based on conformal invariance and its spontaneously symmetry breaking, the purpose of present paper is to study a gravitational model in which *scale transformations* play the key role in obtaining dynamical G and Λ . A scale transformation is different from a conformal transformation. A conformal transformation is viewed as “stretching” all lengths by a space time dependent conformal factor, namely a “unit” transformation. But, a scale transformation is rescaling of metric by a space time dependent conformal factor, and all lengths are assumed to remain unchanged. This kind of transformation is not a “unit” transformation; it is just a dynamical rescaling (enlargement or contraction) of a system. We will take a non-scale invariant gravitational action with a *cosmological constant* in which gravity

couples minimally to a dimensionless dilaton field, and matter couples to a metric which is conformally related, through the dilaton field, to the gravitational metric. Then by a scale transformation, through the dilaton field, we obtain a new action in which gravity couples non-minimally to the dilaton field and matter couples to the gravitational metric. The field equations reveal a cosmological term and a gravitational coupling which are dynamically dependent on the dilaton field (or conformal factor) having a Higgs type potential. The vacuum expectation value of this dilaton field, through spontaneous symmetry breaking on the basis of *anthropic principle*, determines the correlated time variation of G and Λ . The relevance of these time variations to the current acceleration of the universe, together with coincidence problem, Mach's cosmological coincidence and the problems of standard cosmology usually addressed by inflationary models, are discussed.

2 Time variation of G and Λ

We start with the following action¹

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} [\mathcal{R} - 2\bar{\Lambda} - 6g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma] d^4x + S_m(e^{2\sigma} g_{\mu\nu}), \quad (1)$$

where Einstein-Hilbert action with metric $g_{\mu\nu}$ is minimally coupled to a dimensionless dilaton field σ , and the matter is coupled to gravity with the metric $e^{2\sigma} g_{\mu\nu}$ which is conformally related to the metric $g_{\mu\nu}$ ². The parameters $\bar{\kappa}^2$ and $\bar{\Lambda}$ are gravitational coupling and cosmological

¹We will use the sign convention $g_{\mu\nu} = \text{diag}(+, -, -, -)$.

²The idea of coupling matter to conformally related metrics has already been proposed by some authors [5].

constants, respectively. Variation with respect to $g_{\mu\nu}$ and σ yields

$$G_{\mu\nu} + \bar{\Lambda}g_{\mu\nu} = \bar{\kappa}^2 \tilde{T}_{\mu\nu} + \tau_{\mu\nu}, \quad (2)$$

$$\square\sigma = -\frac{\bar{\kappa}^2}{6}\tilde{T}, \quad (3)$$

where

$$\tilde{T}_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m(e^{2\sigma}g_{\mu\nu})}{\delta g^{\mu\nu}}, \quad (4)$$

$$\tau_{\mu\nu} = 6(\nabla_\mu\sigma\nabla_\nu\sigma - \frac{1}{2}g_{\mu\nu}\nabla_\gamma\sigma\nabla^\gamma\sigma), \quad (5)$$

and \tilde{T} is the $g_{\mu\nu}$ trace of the energy-momentum tensor $\tilde{T}_{\mu\nu}$. Now, we introduce the scale transformations

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}, \quad (6)$$

$$\sqrt{-g} \rightarrow \Omega^4 \sqrt{-g}, \quad (7)$$

$$\mathcal{R} \rightarrow \Omega^{-2}\mathcal{R} + 6\Omega^{-3}\nabla_\mu\nabla_\nu\Omega g^{\mu\nu}. \quad (8)$$

where $\Omega = e^{-\sigma}$. The action (1) then becomes

$$S = \frac{1}{2\bar{\kappa}^2} \int \sqrt{-g} [\mathcal{R}\Omega^2 + 6\Omega\square\Omega - 2\bar{\Lambda}\Omega^4 - 6g^{\mu\nu}\nabla_\mu\Omega\nabla_\nu\Omega] d^4x + S_m(g_{\mu\nu}), \quad (9)$$

where gravity couples non-minimally to the dilaton field and matter couples to the gravitational metric $g_{\mu\nu}$. The field equations are obtained by variation of (9) with respect to the fields $g_{\mu\nu}$ and Ω as

$$G_{\mu\nu} + \Omega^2\bar{\Lambda}g_{\mu\nu} = \Omega^{-2}\bar{\kappa}^2 T_{\mu\nu} + \tau_{\mu\nu}(\Omega), \quad (10)$$

and

$$\square\Omega + \frac{1}{12}(\mathcal{R} - 4\bar{\Lambda}\Omega^2)\Omega = 0, \quad (11)$$

where

$$\tau_{\mu\nu}(\Omega) = \Omega^{-1}[3\Box\Omega g_{\mu\nu} - 6\nabla_\mu\nabla_\nu\Omega] + 6\Omega^{-2}[-\frac{1}{2}g_{\mu\nu}\nabla_\gamma\Omega\nabla^\gamma\Omega + \nabla_\mu\Omega\nabla_\nu\Omega]. \quad (12)$$

One may rewrite equation (10) as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu} + \tau_{\mu\nu}(\Omega), \quad (13)$$

where $\Lambda = \Omega^2 \bar{\Lambda}$ and $\kappa^2 = \Omega^{-2} \bar{\kappa}^2$. From Eq.(11) we infer the Higgs type potential for Ω

$$V(\Omega) = \frac{1}{24}(\mathcal{R} - 2\bar{\Lambda}\Omega^2)\Omega^2. \quad (14)$$

For a given negative Ricci scalar³, a positive $\bar{\Lambda}$ leads to vanishing minimum for the conformal factor, namely $\Omega_{min} = 0$. This case is a failure of conformal transformation with zero cosmological constant $\Omega^2 \bar{\Lambda}$, so is not physically viable. The non-vanishing minimum of this potential is obtained for a negative $\bar{\Lambda}$ as

$$\Omega_{min}^2 = \frac{\mathcal{R}}{4\bar{\Lambda}} > 0. \quad (15)$$

Putting this as the vacuum expectation value for Ω in Eq.(13), we obtain

$$G_{\mu\nu} + \frac{\mathcal{R}}{4}g_{\mu\nu} = \frac{4\bar{\Lambda}\bar{\kappa}^2}{\mathcal{R}}T_{\mu\nu}. \quad (16)$$

Equation (16) is the Ω field vacuum condensation of Eq.(13). Considering the Einstein equation in the form

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \tilde{\Lambda} g_{\mu\nu}, \quad (17)$$

where $\tilde{\Lambda} = -\Lambda > 0$, we find that the term $\frac{4\bar{\Lambda}\bar{\kappa}^2}{\mathcal{R}}$ accounts for the dynamical gravitational coupling G , and $-\frac{\mathcal{R}}{4}$ plays the role of a positive dynamical cosmological term $\tilde{\Lambda}$. The negative

³In the sign convention $g_{\mu\nu} = \text{diag}(+, -, -, -)$ the Ricci scalar for Robertson Walker metric is negative.

Ricci scalar asserts to an open universe with $k = -1$. As far as the Ricci scalar evolves dynamically with time, the cosmological term and the gravitational coupling will change in time in a reciprocal way. In other words, $\Lambda\kappa^2$ is an invariant of the scale transformation (5). Therefore, in the early epoch of time evolution of the universe the cosmological term and the gravitational coupling may be very large and very small, in comparison with their current values, respectively. At very late times, however, where the cosmological term is very small the gravitational coupling becomes considerably large, compared with its initial value. This dynamical behavior for the cosmological term and the gravitational coupling is capable of being consistent with the current observations. The obtained value for the cosmological term, namely $\frac{\mathcal{R}}{4}$, is consistent with the upper bound on the current value of cosmological constant: In an expanding Robertson Walker universe (with the radius a , see below) \mathcal{R} decreases with the radius a as $\sim a^{-2}$, so the cosmological term in Eq.(16), (17) fits with its current observational bound provided $\Omega_{min}^2 = a^{-2}$. This form of decaying Λ has already been reported elsewhere [7], [9]. The gravitational coupling also becomes a dynamical quantity with increasing value, in such an expanding universe. It is easy to arrange for such a small value of $\bar{\kappa}^2$ so that $\frac{4\bar{\Lambda}\bar{\kappa}^2}{\mathcal{R}}$ equals to the current value of Newtonian gravitational constant G . In fact, for a late time asymptotic power law expansion of the universe $a(t) \sim t^\alpha$ with $\alpha \geq 1$ the time dependence of G as $\mathcal{R}^{-1} \sim a^2(t) \sim t^{2\alpha}$ leads to

$$\frac{\dot{G}(t)}{G(t)} \sim t^{-1}. \quad (18)$$

This positive time variation of G has already been reported within Kaluza-Klein, Einstein-Yang-Mills, Brans-Dicke and Randall-Sundrum-type models [3], [7].

Time variation of G and Λ has simple explanation in this model: The original action (1)

contains dimensional constants $\bar{\Lambda}, \bar{\kappa}^2$. By a scale transformation we obtain the new action (9) with dynamical terms Λ, κ^2 . Finally, using the vacuum expectation value of the Ω field we obtain the desired time variation for $\Lambda(t), \kappa^2(t)$. If we were to impose conformal transformation instead of scale transformation then $\bar{\Lambda}, \bar{\kappa}^2$ would also conformally transform according to their dimensions as

$$\bar{\Lambda} \rightarrow \Omega^{-2} \bar{\Lambda}, \quad \bar{\kappa}^2 \rightarrow \Omega^2 \bar{\kappa}^2. \quad (19)$$

Eqs.(10), (11) would then become

$$G_{\mu\nu} + \bar{\Lambda} g_{\mu\nu} = \bar{\kappa}^2 T_{\mu\nu} + \tau_{\mu\nu}(\Omega), \quad (20)$$

and

$$\square \Omega + \frac{1}{12}(\mathcal{R} - 4\bar{\Lambda})\Omega = 0, \quad (21)$$

respectively, which apparently are very different from their original forms (13), (14) with completely different physics. In fact, there is no non-trivial vacuum expectation value for Ω except $\Omega_{min} = 0$. Therefore, unlike other models [8], in this model only scale transformation can provide the desired features we are looking for, such as dynamical Λ and κ^2 .

Vacuum condensation of Ω field to Ω_{min} is of particular importance. As far as $\bar{\Lambda} > 0$, there is no spontaneous symmetry breaking and vacuum condensation. It happens once $\bar{\Lambda}$ becomes negative. But $\bar{\Lambda}$ is assumed to be a fundamental constant in the model and can not change from a positive to a negative value. To resolve this problem one may resort to the *anthropic principle*. According to this principle, what we perceive as the cosmological constant is in fact a stochastic variable which varies on a larger structure (a multi-verse), and takes different values in different universes. Therefore, we live in a universe in which the cosmological constant

is compatible with the development of life. Using this approach in the present model, one may assume that only those universes with $\bar{\Lambda} < 0$ are capable of condensating the dilaton field to its vacuum expectation value. Other universes with $\bar{\Lambda} > 0$ are then ruled out by anthropic principle. This mechanism now works like spontaneous symmetry breaking so that discarding $\bar{\Lambda} > 0$ universes in favor of $\bar{\Lambda} < 0$ ones, according to anthropic principle, acts like a selection rule that causes $\bar{\Lambda}$ to play the role of an *effective* order parameter changing from positive to negative values, and leading to vacuum condensation of dilaton field in $\bar{\Lambda} < 0$ universes.

3 Acceleration of the universe

We take $g_{\mu\nu}$ and $T_{\mu\nu}$ to be the Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t) \left(\frac{dr^2}{(1 - kr^2)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (22)$$

and perfect fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (23)$$

respectively, where $k = 0, \pm 1$ and a is the scale factor. Substituting $g_{\mu\nu}$ and $T_{\mu\nu}$ into the Einstein equation (17), provided $\tilde{\Lambda} = -\bar{\Lambda}\Omega_{min}^2 = -\bar{\Lambda}a^{-2}$, we obtain the following field equations

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} + \frac{\bar{\Lambda}}{3a^2} = \frac{1}{3}\bar{\kappa}^2 a^2 \rho, \quad (24)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} + \frac{\bar{\Lambda}}{a^2} = -\bar{\kappa}^2 a^2 p, \quad (25)$$

where \dot{a} means time derivative with respect to ct . Combining Eqs.(24) and (25) we obtain the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\bar{\kappa}^2 a^2 \left(\frac{1}{3}\rho + p \right) - \frac{\bar{\Lambda}}{3a^2}, \quad (26)$$

and the conservation equation

$$\dot{\rho} = -3\frac{\dot{a}}{a} \left[\left(\frac{5}{3}\rho + p \right) + \frac{2\bar{\Lambda}}{3\bar{\kappa}^2 a^4} \right]. \quad (27)$$

If we put the power law behavior $\rho = Aa^\alpha$ and equation of state $p = \omega\rho$ into Eq.(27) the density and pressure of the perfect fluid are obtained

$$\rho = -\frac{2}{1+3\omega}\bar{\kappa}^{-2}\bar{\Lambda}a^{-4}, \quad (28)$$

$$p = -\frac{2\omega}{1+3\omega}\bar{\kappa}^{-2}\bar{\Lambda}a^{-4}. \quad (29)$$

These results are novel in that the behavior of the density ρ and pressure p in terms of the scale factor a is the same a^{-4} regardless of the equation of state parameter ω . In fact, unlike the usual standard cosmology where the equation of state plays the role of setting the power of the scale factor, it just sets the multiplicative factors $-\frac{2}{1+3\omega}\bar{\kappa}^{-2}\bar{\Lambda}$, $-\frac{2\omega}{1+3\omega}\bar{\kappa}^{-2}\bar{\Lambda}$ in this model, and leaves the power law behavior $\sim a^{-4}$ to be the same for any equation of state. This is surprising, and tells us that all types of matter including radiation show the same behavior with respect to the scale factor, and this brings new opportunities to solve some cosmological problems, as will be discussed later.

We now put Eqs.(28), (29) into the acceleration equation (26) which leads to

$$\frac{\ddot{a}}{a} = 0, \quad (30)$$

for each constant value of ω . The energy equation (24) with $k = -1$ ($\mathcal{R} < 0$) becomes

$$\dot{a}^2 = 1 - \frac{\bar{\Lambda}}{3} \left(1 + \frac{2}{1+3\omega} \right), \quad (31)$$

which shows a constant value of \dot{a} for each constant parameter ω . Integration of this equation,

in terms of the time parameter ct leads to

$$a(t) = a(0) + \sqrt{1 - \frac{\bar{\Lambda}}{3}(1 + \frac{2}{1 + 3\omega})}ct. \quad (32)$$

Comparing the radiation ($\omega = 1/3$) and dust ($\omega = 0$) parameters, we realize that

$$\rho_{Radiation} < \rho_{Dust}$$

for a given scale factor. On the other hand,

$$\dot{a}_{Radiation} < \dot{a}_{Dust}.$$

This is very interesting because it predicts an acceleration during the phase transition from radiation to dust eras. The energy equation also shows interesting features for the early universe: It is shown by quantum cosmological consideration that the decaying cosmological term $\Lambda \sim a^{-2}$ may have its origin in the cosmic strings as a type of exotic matter with the effective equation of state $p \approx -\frac{1}{3}\rho$ [9]. If we assume such an exotic type of matter would exist in the very early universe with $\omega \approx -\frac{1}{3}$, then \dot{a} would become such a huge velocity which could solve the problems of standard cosmology, such as horizon, flatness and magnetic monopole problems, in a way similar to the inflationary models.

If ω would be a continuously varying parameter, the deceleration parameter would then result in

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\bar{\Lambda}a\dot{\omega}}{\dot{a}^3(1 + 3\omega)^2}. \quad (33)$$

However, ω has not such a continuous feature; instead it changes somehow during phase transitions. We suppose some typical time variation of the parameter ω during different phase transitions. Then, for example, during phase transition from some exotic type matter era

($\omega \approx -\frac{1}{3}$) toward the radiation era ($\omega \approx \frac{1}{3}$) we have $\delta\omega > 0$ and $q > 0$ which accounts for a decelerating universe. On the other hand, a phase transition from the radiation ($\omega \approx \frac{1}{3}$) toward dust ($\omega \approx 0$) eras leads to $\delta\omega < 0$ and $q < 0$ which shows an accelerating universe.

4 Coincidence problem and Mach's cosmological coincidence

It is usually understood, in the standard cosmology, that the matter density scales with the expansion of the universe as

$$\rho_M \sim \frac{1}{a^3},$$

and the vacuum energy density ρ_V is almost constant. So, there is only one epoch in the history of the universe when $\rho_M \sim \rho_V$. It is difficult to understand why we happen to live in this special epoch. In other words: How finely-tuned is it that we exist in the era when vacuum and matter are comparable? This is known as *Coincidence problem* [10].

In the present model, the energy density of the vacuum is given by $\rho(t) = \Lambda(t)/8\pi G(t)$. Keeping the dimensions of all quantities, suppose we take $-\bar{\Lambda} \sim 0.1 - 1$. Then, Ω_{min} behaves numerically as $\simeq a^{-2}$ which results in the desired behavior $\Lambda(t) \simeq a^{-2}$, in good agreement with observational bound on the current value of the cosmological constant. On the other hand, $G(t) \sim \bar{\kappa}^2 a^2$. Therefore, we obtain the following relation

$$\rho_V \sim \frac{1}{\bar{\kappa}^2 a^4}.$$

Unlike the behavior $\rho_M \sim a^{-3}$ in the standard cosmology, the matter density in the matter

dominated era $\omega \approx 0$, according to (28), scales with the expansion of the universe as

$$\rho_M = -\frac{2\bar{\Lambda}}{\bar{\kappa}^2 a^4},$$

which has the same scale factor dependence as that of the vacuum energy density ρ_V . Now, demanding $\frac{\rho_M}{\rho_V} \approx \frac{0/030}{0/070}$, to account for the current observation, requires simply

$$\bar{\Lambda} \approx -\frac{3}{14}.$$

Therefore, we come to an important conclusion: There is no a real coincidence problem. It is just a result of $\rho_M \sim a^{-4}$ law, in the present model.

This law, if combined with the mass definition

$$\rho_M \sim \frac{M}{a^3},$$

leads to

$$M \simeq \frac{1}{\bar{\kappa}^2 a}.$$

But, using $G \simeq \bar{\kappa}^2 a^2$, this relation is cast in the form of the well-known *cosmological coincidence* usually referred to Mach

$$\frac{GM}{a} \sim 1.$$

Therefore, the relation (not coincidence !) $\rho_M \sim \rho_V$ is cast in the form of Mach's *cosmological coincidence* $\frac{GM}{a} \sim 1$. In a reciprocal way, Mach's *cosmological coincidence* is nothing but a simple result of the $\rho_M \sim a^{-4}$ law.

Conclusion

In this paper, we have studied a model of scale transformation imposed on a gravitational action which is not scale invariant due to the presence of dimensional gravitational and cosmological constants. By choosing a dilaton field for scale transformation, we obtained a *new* action whose field equations revealed dynamical cosmological term and gravitational coupling. Unlike other approaches [8] in which the dynamics of Λ (or perhaps G) is obtained by resorting to a conformal invariant action where dynamical distinction between two unit systems in cosmology and particle physics play a key role, in the present approach the dynamics of Λ and G is not due to any scale invariance. Instead, in this non-scale invariant theory the dynamics of Λ and G is obtained by introducing a dynamical scale through a dilaton field. Fixing this scale, through the vacuum expectation value of dilaton field, leads to time variations of Λ and G . These time variations lead the vacuum energy density to be time dependent as $\rho_v = \Lambda(t)/8\pi G(t)$. Therefore, at early universe where Λ is so large and G is too small compared with their current values, the vacuum energy is huge. At the present status of the universe, however, the vacuum energy is vanishing, like a^{-4} , due to time variations of both Λ and G . This solves the cosmological constant problem in the present model.

We have also studied the relevance of this model to the other problems of standard cosmology such as current acceleration of the universe, coincidence problem and those usually addressed by inflationary models. We have shown that there is no principal competition between ρ_v , ρ_M to account for the current acceleration of the universe. Instead, the important role is played by the variation of ω , the parameter in the equation of state, from radiation ($\omega \approx \frac{1}{3}$) toward dust ($\omega \approx 0$) eras, which leads to an accelerating universe. Removing the

competition between ρ_v , ρ_M and realizing the variation of $\omega_R \approx \frac{1}{3}$ towards $\omega_M \approx 0$ as the reason for the current acceleration of the universe, the coincidence problem fades away. In fact, there is no a direct relation between the acceleration of the universe and coincidence problem in the present model. The acceleration of the universe owes to the variation of ω from radiation toward dust eras and has nothing to do with such coincidence as $\rho_M \sim \rho_V$. Moreover, there is no coincidence problem in this model, at all. This is because, all the matter, radiation and vacuum densities goes like a^{-4} and there is no important competition between them, in this regard.

However, the model is very sensitive to the equation of state with ($\omega \approx -\frac{1}{3}$) which might have been occurred by an exotic type matter, namely cosmic strings, at very early universe. This value of ω may cause such a huge velocity for the scale factor that can resolve the well-known problems (horizon, flatness, etc.) usually addressed by inflationary models. It is worth noticing that the vacuum energy density ρ_v seems not to play direct role both in the super fast expansion of the early universe (due to an exotic matter) and current acceleration of the universe (due to phase transition from radiation toward dust eras). However, according to (31) and (32), if ρ_v would be zero, namely $\bar{\Lambda} = 0$, neither super fast expansion of the early universe nor acceleration of the present universe would then occur.

We have shown that the apparent coincidences $\rho_M \sim \rho_V$ and $\frac{GM}{a} \sim 1$ are not really coincidences. They are just simple results of $\rho \sim a^{-4}$ behavior. This behavior leads to

$$M \sim \frac{1}{\bar{\kappa}^2 a},$$

for which we obtain

$$\frac{\dot{M}}{M} = -\frac{\dot{a}}{a} = -H,$$

where H is the Hubble constant. It is important to note that the time variation of M is not a problem at all, and is consistent with the conservation equation (27). So, how can we physically interpret the behavior $\rho \sim a^{-4}$ or $M \sim a^{-1}$ for the observable matter M ? This may be answered from different viewpoints. For example, in the space-time-matter interpretation of noncompact Kaluza-Klein theories the observable mass is time dependent as [11]

$$\frac{\dot{M}}{M} = -\frac{\mathcal{A}}{t}.$$

Another possible explanation for the behavior $\rho_M \sim a^{-4}$ lies in the principal duality between matter and radiation from quantum mechanical point of view. In the same way as the radiation density behaves like $\rho_R \sim a^{-4}$ due to an extra effect of red shift in an expanding universe, one may assume the same red shift effect occurs somehow for de Broglie waves of matter and leads to $\rho_M \sim a^{-4}$.

Finally, we point out that the solution (32) for the scale factor becomes free of singularity provided we take $a(0)$, for example, to be a nonzero scale of Planck length.

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